RF Cavities and beam loading compensation
in the Compressor of a Proton Driver

1. Cavity parameters
   a. ACOL cavity

   The bunch rotation cavity in the ACOL project [1] has the following characteristics:

   \[
   \frac{R}{Q} = 39 \, \Omega \\
   f_R = 9.55 \text{ MHz} \\
   Q = 10980
   \]

   The inductance \( L \), capacitance \( C \) and shunt resistance \( R_S \) of the equivalent resonant circuit can be derived from the following equations:

   \[
   \frac{R_S}{Q} = \sqrt{\frac{L}{C}} \\
   f_R = \frac{1}{2\pi \sqrt{LC}}
   \]

   which gives:

   \[
   L = 0.65 \, \mu\text{H} \\
   C = 427 \, \text{pF} \\
   R_S = 428.2 \, \text{k}\Omega
   \]

   That can be obtained (see Annex 1) with gap disks of 3.6 m\(^2\) area separated by 0.15 m.

   b. Compressor cavity (6 bunch scheme described in ref. [2])

   To reduce the resonant frequency to:

   \[
   f'_R = 2.826 \text{ MHz}
   \]

   the \( LC \) product must be multiplied by \( \left( \frac{f_R}{f'_R} \right)^2 = 11.42 \).

   In practice the diameter \( D \) of the cavity can be slightly increased to 3 m (instead of 2.5 m) while the overall length will not be changed. Assuming that the inductance scales like the diameter, it results that:
Therefore

\[ R' \frac{Q'}{Q} = 13.9 \Omega \]

A summary of all the main characteristics of the Compressor cavity is shown in Table 1. Details about their derivation are given in Annex 2.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resonance frequency</td>
<td>2.826 MHz</td>
</tr>
<tr>
<td>Normalised impedance</td>
<td>13.9 Ω</td>
</tr>
<tr>
<td>Equivalent inductance</td>
<td>0.78 μH</td>
</tr>
<tr>
<td>Equivalent capacitance</td>
<td>4.06 nF</td>
</tr>
<tr>
<td>Shunt resistance</td>
<td>120 kΩ</td>
</tr>
<tr>
<td>Quality factor</td>
<td>8600</td>
</tr>
<tr>
<td>Filling time</td>
<td>970 μs</td>
</tr>
</tbody>
</table>

### Table 1: main characteristics of the Compressor cavity

2. **Beam loading compensation**

a. **Principle**

The cavity is supposed to be operated at full RF field before the beam starts circulating in the Compressor. The RF amplifier is matched to the cavity to optimize the transfer of RF power:
- a power of 167 kW to obtain an RF voltage of 200 kV_{peak} across the gap.
- the voltage will be established after a few ms (it can be made shorter if the amplifier can provide more than the strictly necessary power).

These design choices minimize cost. They imply however that it will be impossible to actively compensate for beam induced transients, since the beam will circulate in the Compressor for an order of magnitude less time than the time constant of the amplifier-cavity set-up (< 100 μs compared to 970 μs).

Before the beam enters the Compressor, an RF field will be established in the cavity, at the frequency of the RF generator (\( \omega_G = 2\pi f_G \)):

\[ V_G = \tilde{V} e^{j\omega_G t} \]

Approximating it with a Dirac pulse, a bunch of charge q passing through the cavity gap will develop an instantaneous voltage \( \Delta V_B \) given by:

\[ \Delta V_B = \frac{q}{C} \]
It will trigger a damped oscillation at the cavity resonant frequency ($\omega_R = 2\pi f_R$), beginning in quadrature with the generator induced voltage:

$$\Delta V_B = \Delta V_B e^{j(\omega_R t + \frac{\pi}{2}) - t/\tau}$$

Damping can be neglected because of the short duration of the compression process with respect to the cavity filling time.

The voltage in the cavity $V_T$ will result from the sum of the voltages induced by the generator and the beam. Assuming that the cavity is tuned at the beam frequency, the beam induced voltage will increase linearly with time. The result, if the RF generator is driving the cavity at the same frequency, is represented in Figure 1, in a frame rotating at the beam frequency. If $\Delta V_B$ is non negligible, the total voltage will rotate in phase which will disturb the bunch rotation process.

To mitigate this effect, the proposal is to drive the cavity with the RF generator at a frequency which differs from the beam, so that the total voltage stays approximately at constant phase with respect to the beam (Figure 2). Although the result is better, the process is still imperfect because the amplitude of the total voltage will clearly be reduced after a few bunch passages.

A better result can be obtained tuning the cavity at a different frequency, so that the beam induced voltage will be also rotating in phase with respect to the beam, as illustrated in Figure 3.
b. First order application

Each bunch in the compressor has $1.7 \times 10^{13}$ protons, which corresponds to $2.72 \, \mu\text{Coulomb}$. The induced voltage per passage in the cavity is

$$\Delta V_B = 670 \, V$$

At the end of rotation of a bunch in the middle of the bunch train, if all frequencies are equal (case of Figure 1), the total induced voltage will be

$$3 \times 36 \times \Delta V_B = 72 \, kV$$

It is significant with respect to the nominal cavity voltage of 200 kV, and shows the need for mitigation measures.

The scheme described in Figure 2 necessitates a phase shift of $\delta \Phi_G$ of the generator induced voltage with respect to the beam frequency at each period.

$$\delta \Phi_G = \frac{\Delta V_B}{V_G} = 3.35 \, \text{mrad}$$

This can be obtained with a frequency difference of $0.533 \times 10^{-3}$ or 1.5 kHz. At the end of rotation, the generator induced voltage will have rotated by 0.361 rad, which will induce approximately a 6.5 % change in the amplitude and 7 mrad in the phase of the total voltage.

c. Real case

In reality the beam circulating in the Compressor will not be immediately made up of three bunches, and all bunches will not be ejected simultaneously. It is therefore necessary to model the real sequence of beam injections and ejections to correctly simulate the effect on the cavity voltage and, ultimately, quantify the quality of bunch rotation in the longitudinal phase space.

References

ANNEX 1: Derived parameters of the ACOL cavity

For the complete ACOL cavity:

\[ C = \varepsilon_0 \frac{2S}{d} \]

Where \( S \) is the surface of one disk and \( d \) the distance between the disk.

Estimating that:

\[ S = \pi (1.1^2 - 0.25^2) = 3.6 \text{ m}^2 \]

Since \( C = 427 \text{ pF} \), it implies that:

\[ d = 0.149 \text{ m} \]

Therefore the gradient in the ACOL cavity is:

\[ E = 0.75/0.15 = 5 \text{ MV/m} \]

which is approximately the Kilpatrick limit at 9.5 MHz.
ANNEX 2 : Characteristics of the Compressor cavity

- **Size**

With respect to the ACOL cavity and assuming that the inductance can be enlarged by a factor 1.2 using a larger diameter, the capacitance must be increased by a factor 9.517.

The Kilpatrick limit for the gradient is however reduced by a factor 0.74 (from 5.3 MV/m at 9.6 MHz to 3.9 MV/m at 2.8 MHz).

The goal being at 100 kV/m (real estate gradient) at 2.8 MHz, while it was 375 kV/m at 9.6 MHz, the distance between gap disks can only be reduced by the factor:

\[
\frac{1}{0.74} \cdot \frac{100}{375} = 0.36
\]

which will increase \(C\) by the inverse factor 2.775.

Doubling the number of capacitive disks will multiply \(C\) by a factor of 2.

The missing factor (1.714) has to result from the increased area of the disk. This is feasible if the external diameter of the gap disks is brought up to 1.416 m:

\[
S' = \pi(1.416^2 - 0.2^2) = 6.17 \text{ m}^2 = 1.714 \times S
\]

Starting from these dimensions, electromagnetic codes (e.g. SuperFish) has to be used to refine the cavity geometry.

- **Other electrical characteristics**

The dissipation in the cavity \(P\) is mostly due to the heat dissipated by the current \(I_L\) circulating in the resistance \(R_L\) in series with the inductance:

\[
P = R_L I_L^2
\]

For a voltage \(V\) in the resonator, \(P\) can also be expressed as a function of the shunt resistance \(R_S\):

\[
P = \frac{V^2}{R_S}
\]

Hence

\[
R_S = \frac{1}{R_L} \left( \frac{V}{I_L} \right)^2 = \frac{1}{R_L} (L \omega R)^2 = \frac{1}{R_L} \left( \frac{R}{Q} \right)^2
\]
Assuming that the resistivity of the material is independent of frequency, the resistance $R_L$ depends upon the depth of penetration of the current which is inversely proportional to $\omega^{1/2}$ and the transverse dimension $D$ of the cavity with the following relation:

$$R_L \propto \frac{\omega^{1/2}}{D}$$

From these relations it results that:

$$R_S \propto \frac{D}{\omega^{1/2}} \left(\frac{R}{Q}\right)^2$$

The ratio of the shunt resistance $R'_S$ of the Compressor cavity to the shunt resistance $R_S$ of the ACOL cavity is then:

$$\frac{R'_S}{R_S} = \frac{1.2}{0.296^{0.5}} (0.356)^2 \approx 0.28$$

Hence

$$R'_S \approx 120 \, k\Omega$$

and

$$Q' \approx 8600$$

The filling time constant $\tau'$ of the Compressor cavity is then:

$$\tau' = \frac{2Q'}{\omega'_R} \approx 970 \, \mu s$$