Cyclotrons
accelerator school – introductory course
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Cyclotrons I - Outline

- **the classical cyclotron**
  - history of the cyclotron, basic concepts and scalings, focusing, stepwidth, relativistic relations, classification of cyclotron-like accelerators
- **synchro-cyclotrons**
  - concept, synchronous phase, example
- **isochronous cyclotrons** (→ sector cyclotrons)
  - isochronous condition, focusing in Thomas-cyclotrons, spiral angle, classical extraction: pattern/stepwidth, transverse and longitudinal space charge
The Classical Cyclotron

- two capacitive electrodes „Dees“, two gaps per turn
- internal ion source
- homogenous B field
- constant revolution time
  (for low energy, $\gamma \approx 1$)

$$\omega_c = \frac{eB_z}{m}$$

powerful concept:
- simplicity, compactness
- continuous injection/extraction
- multiple usage of accelerating voltage
first cyclotron: 1931, Berkeley
1kV gap-voltage 80keV Protons

Lawrence & Livingston, 27inch Zyklotron

Ernest Lawrence, Nobel Prize 1939
PSI Ring Cyclotron - 1974
cyclootron frequency and $K$ value

- **cyclootron frequency** (homogeneous) B-field:
  \[ \omega_c = \frac{eB}{\gamma m_0} \]

- **cyclootron $K$-value**:
  \[ K = \frac{e^2}{2m_0} (B\rho)^2 \]
  → $K$ is the **kinetic energy reach** for protons **from bending strength** in non-relativistic approximation.

  → $K$ can be used to rescale the energy reach of protons to other charge-to-mass ratios:
  \[ \frac{E_k}{A} = K \left( \frac{Q}{A} \right)^2 \]

  → $K$ in [MeV] is often used for naming cyclotrons

  examples:  K-130 cyclotron / Jyväskylä
cyclone C230 / IBA
cyclotron - isochronicity and scalings

continuous acceleration → revolution time should stay constant, though $E_k$, $R$ vary

magnetic rigidity:

$$BR = \frac{p}{e} = \beta \gamma \frac{m_0 c}{e}$$

orbit radius from isochronicity:

$$R = \frac{c}{\omega_c} \beta = R_\infty \beta$$

$$= \frac{c}{\omega_c} \sqrt{1 - \gamma^{-2}}$$

deduced scaling of $B$:

$$R \propto \beta; \quad BR \propto \beta \gamma \quad \rightarrow \quad B(R) \propto \gamma(R)$$

thus, to keep the isochronous condition, $B$ must be raised in proportion to $\gamma(R)$; this contradicts the focusing requirements!

technical solutions discussed under sector cyclotrons
the field index describes the (normalized) radial slope of the bending field:

\[ n = -\frac{R}{B} \frac{dB}{dR} = -\frac{\beta}{\gamma} \frac{d\gamma}{d\beta} = 1 - \gamma^2 \]

from isochronous condition: \( B \propto \gamma, \ R \propto \beta \)

→ thus \( n < 0 \) (positive slope of field) to keep beam isochronous!
cyclootron stepwidth classical (nonrelativistic)

\[ m\ddot{R} = m\frac{v^2}{R} - qvB_z = 0 \]
\[ qRB_z = \sqrt{2mE_k} \]
\[ \frac{dR}{R} = \frac{1}{2} \frac{dE_k}{E_k} \]

equation of motion for ideal centroid orbit \( R, \rightarrow \) relation between energy and radius

use:

\[ \Delta E_k = \text{const}; B_z = \text{const}; E_k \propto R^2 \]

thus:

\[ \Delta R \propto \frac{R}{E_k} \propto \frac{1}{R} \]

radius increment per turn decreases with increasing radius \( \rightarrow \) extraction becomes more and more difficult at higher energies

"cyclootron language"
focusing in a classical cyclotron

centrifugal force $\frac{mv^2}{r}$

Lorentz force $qv \times B$

$$m\ddot{x} = m r \dot{\theta}^2 - q r \dot{\theta} B_z$$

focusing: consider small deviations $x$ from beam orbit $R$ ($r = R + x$):

$$\ddot{x} + \frac{q}{m} v B_z (R + x) - \frac{v^2}{R + x} = 0,$$

$$\ddot{x} + \frac{q}{m} v \left(B_z(R) + \frac{dB_z}{dR} x\right) - \frac{v^2}{R} \left(1 - \frac{x}{R}\right) = 0,$$

$$\ddot{x} + \omega_c^2 (1 - n)x = 0.$$

using: $\omega_c = q B_z / m = v / R$, $r \dot{\theta} \approx v$, $n = -\frac{R}{B} \frac{dB}{dR}$
betatron tunes in cyclotrons

thus in radial plane:

\[
\begin{align*}
\omega_r &= \omega_c \sqrt{1 - n} = \omega_c \nu_r \\
\nu_r &= \sqrt{1 - n} \\
&\approx \gamma
\end{align*}
\]

note: simple case for \( n = 0 \): \( \nu_r = 1 \)
(one circular orbit oscillates w.r.t the other)

using Maxwell to relate \( B_z \) and \( B_R \):

\[
\text{rot } \vec{B} = \frac{dB_R}{dz} - \frac{dB_z}{dR} = 0
\]

in vertical plane:

\[\nu_z = \sqrt{n}\]

\( n > 0 \) to obtain vertical focus.

thus: in classical cyclotron \( n > 0 \) required;
however this violates isochronous condition \( n = 1 - \gamma^2 < 0 \)
relativistic quantities in the context of cyclotrons

**Energy**

\[ E = \gamma E_0 \]

**Kinetic Energy:**

\[ E_k = (\gamma - 1)E_0 \]

**Velocity**

\[ v = \beta c \]

**Momentum**

\[ p = \beta \gamma m_0 c \]

**Revolution Time:**

\[ \tau = \frac{2\pi R}{\beta c} \]

**Bending Strength:**

\[ BR = \beta \gamma \frac{m_0 c}{e} \]

**Numerical Example for Protons**

<table>
<thead>
<tr>
<th>( E_k ) [MeV]</th>
<th>( \gamma )</th>
<th>( \beta )</th>
<th>( p ) [MeV/c]</th>
</tr>
</thead>
<tbody>
<tr>
<td>590</td>
<td>1.63</td>
<td>0.79</td>
<td>1207</td>
</tr>
</tbody>
</table>

Compare surface Muons: \( p=29.8\text{MeV/c} \rightarrow 40\) times more sensitive than \( p_{590\text{MeV}} \) in same field
useful for calculations – differential relations

\[
\frac{d\beta}{\beta} = \frac{1}{\gamma (\gamma + 1)} \frac{dE_k}{E_k}
\]

\[
\frac{dE_k}{E_k} = \frac{\gamma + 1}{\gamma} \frac{dp}{p}
\]

Example: speed gain per turn in a cyclotron; comparison to classical \(mv^2/2\)

<table>
<thead>
<tr>
<th>(E_k)</th>
<th>(\Delta E_k / \text{turn})</th>
<th>(\Delta \beta / \beta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>590MeV</td>
<td>3.4MeV</td>
<td>1.3%o</td>
</tr>
<tr>
<td>classical calculation</td>
<td></td>
<td>(2.9%o)</td>
</tr>
</tbody>
</table>
concepts of cyclotrons to establish ...

1.) resonant acceleration

- limit energy / ignore problem [classical cyclotron]
- frequency is varied [synchro- cyclotron]
- avg. field slope positive [isochronous cyclotron]

2.) transverse focusing

- negative field slope [classical cyclotron]
- focusing by flutter, spiral angle [AVF/sector cyclotron]
classification of cyclotron like accelerators

- **classical cyclotron**
  \[ B(\theta) = \text{const} \]

- **Thomas cyclotron**
  [Azimuthally Varying Field, e.g. \( B(\theta) \propto b + \cos(3\theta) \), one pol]

- **separated sector cyclotron**
  [separated magnets, resonators]

- **synchro-cyclotron**
  [varying RF frequency]

- **Fixed Focus Alternating Gradient Accelerator (FFAG)**
  [varying RF, strong focusing]

**AVF concept – harmonic pole shaping, electron model, Richardson et al (1950), courtesy of Lawrence Berkeley National Laboratory**

- **high intensity**
- **high energy**
- **compact machine**
next: synchro-cyclotrons

- concept and properties
- frequency variation and synchronous phase
- an example for a modern synchrocyclotron
Synchrocyclotron - concept

- accelerating frequency is variable, is reduced during acceleration
- positive field index (= negative slope) ensures sufficient focusing
- operation is pulsed, thus avg. intensity is low
- bending field constant in time, thus rep. rate high, e.g. 1kHz

first proposal by Mc.Millan, Berkeley
Synchrocyclotron continued

### Numerical Example

<table>
<thead>
<tr>
<th>Field and Frequency vs. Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 230MeV p, strong field</td>
</tr>
<tr>
<td>- RF curve must be programmed</td>
</tr>
</tbody>
</table>

### Advantages

- High energies possible (≥1Gev)
- Focusing by field gradient, no complicated flutter required → thus compact magnet
- Only RF is cycled, fast repetition as compared to synchrotron

### Disadvantages

- Low intensity, at least factor 100 less than CW cyclotron
- Complicated RF control required
- Weak focusing, large beam

![Graph showing frequency and field strength vs. radius](image)
Synchrocyclotron and synchronous phase

- Internal source generates continuous beam; only a fraction is captured by RF wave in a phase range around a synchronous particle.
- In comparison to a synchrotron, the "storage time" is short, thus in practice no synchrotron oscillations.

\[
\frac{qU_0N \cos \varphi_s}{E_k + E_0} = -\frac{2\pi}{\omega^2} \frac{d\omega}{dt}
\]
A modern synchrocyclotron for medical application – IBA S2C2

→ at the same energy synchrocyclotrons can be build more compact and with lower cost than sector cyclotrons; however, the achievable current is significantly lower

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>energy</td>
<td>230 MeV</td>
</tr>
<tr>
<td>current</td>
<td>20 nA</td>
</tr>
<tr>
<td>dimensions</td>
<td>∅2.5 m x 2 m</td>
</tr>
<tr>
<td>weight</td>
<td>&lt; 50 t</td>
</tr>
<tr>
<td>extraction radius</td>
<td>0.45 m</td>
</tr>
<tr>
<td>s.c. coil strength</td>
<td>5.6 Tesla</td>
</tr>
<tr>
<td>RF frequency</td>
<td>90...60 MHz</td>
</tr>
<tr>
<td>repetition rate</td>
<td>1 kHz</td>
</tr>
</tbody>
</table>
compact treatment facility using the high field synchro-cyclotron

- required area: 24x13.5m² (is small)
- 2-dim pencil beam scanning
• next: isochronous- / sector cyclotrons
  – focusing and AVF vs. separated sector cyclotron
  – how to keep isochronicity
  – extraction: pattern/stepwidth
  – RF acceleration
  – transv./long. space charge
focusing in sector cyclotrons

hill / valley variation of magnetic field (Thomas focusing) makes it possible to design cyclotrons for higher energies

Flutter factor:

\[ F^2 = \frac{B_z^2 - B_z'^2}{B_z^2} \]

with flutter and additional spiral angle of bending field:

\[ \nu_z^2 = -\frac{R}{B_z} \frac{d B_z}{d R} + F^2 (1 + 2 \tan^2 \delta) \]

strong term

\[ e.g.: \delta = 27^\circ: \quad 2\tan^2\delta = 1.0 \]
Azimuthally Varying Field vs. Separated Sector Cyclotrons

- AVF = single pole with shaping
- often spiral poles used
- internal source possible
- D-type RF electrodes, rel. low energy gain
- compact, cost effective
- depicted Varian cyclotron: 80% extraction efficiency; not suited for high power

- modular layout, larger cyclotrons possible, sector magnets, box resonators, stronger focusing, injection/extraction in straight sections
- external injection required, i.e. pre-accelerator
- box-resonators (high voltage gain)
- high extraction efficiency possible:
  e.g. PSI: $99.98\% = (1 - 2 \cdot 10^{-4})$
three methods to raise the average magnetic field with $\gamma$

1.) broader hills (poles) with radius
2.) decrease pole gap with radius
3.) strong s.c. coil leads to field enhancement at large radius

remember:

rev.time: $R \propto \beta$
momentum: $BR \propto \beta \gamma$
thus: $B \propto \gamma$

(photo: S. Zaremba, IBA)
field stability is critical for isochronicity
example: medical Comet cyclotron (PSI)

\[
\Delta \phi_{RF} \propto n_{\text{turn}} \frac{\Delta B}{B}
\]
e.g.: \( n_{\text{turn}} = 600 \)

Current in main coil (A)

158.41  158.43  158.45
derivation of turn separation in a cyclotron

starting point: bending strength
→ compute total log.differential
→ use field index $k = R/B \cdot dB/dR$

\[
BR = \sqrt{\gamma^2 - 1} \frac{m_0c}{e}
\]

\[
\frac{dB}{B} + \frac{dR}{R} = \frac{\gamma d\gamma}{\gamma^2 - 1}
\]

\[
\frac{dR}{d\gamma} = \frac{\gamma R}{\gamma^2 - 1} \frac{1}{1 - n}
\]

radius change per turn

\[
\frac{dR}{d\eta_t} = \frac{dR}{d\gamma} \frac{d\gamma}{d\eta_t}
\]

\[
= \frac{U_t}{m_0c^2} \frac{\gamma R}{(\gamma^2 - 1)(1 - n)}
\]

\[
= \frac{U_t}{m_0c^2} \frac{R}{(\gamma^2 - 1)\gamma}
\]

\[U_t = \text{energy gain per turn}\]

\{ \text{isochronicity not conserved (last turns)} \}

\{ \text{isochronicity conserved (general scaling)} \}
turn separation - discussion

for clean extraction a large stepwidth (turn separation) is of utmost importance; in the PSI Ring most efforts were directed towards maximizing the turn separation.

general scaling at extraction:

\[ \Delta R(R_{\text{extr}}) = \frac{U_t}{m_0 c^2} \left( \frac{R_{\text{extr}}}{\gamma^2 - 1} \right) \gamma \]

scaling during acceleration:

\[ \frac{dR}{dn_t} \approx \frac{U_t}{m_0 c^2} \frac{R}{\beta^2} \rightarrow \Delta R(R) \propto \frac{1}{R} \]

illustration: stepwidth vs. radius in cyclotrons of different sizes but same energy; 100MeV inj \(\rightarrow\) 800MeV extr

Desirable:
- limited energy (< 1GeV)
- large radius \(R_{\text{extr}}\)
- high energy gain \(U_t\)
extraction with off-center orbits

betatron oscillations around the “closed orbit” can be used to increase the radial stepwidth by a factor 3!

radial tune vs. energy (PSI Ring)
typically $\nu_r \approx \gamma$ during acceleration; but decrease in outer fringe field

without orbit oscillations: stepwidth from $E_k$-gain (PSI: 6mm)

with orbit oscillations: extraction gap; up to 3 x stepwidth possible for $\nu_r=1.5\pi$ (phase advance)

beam to extract

phase vector of orbit oscillations $(r,r')$
extraction profile measured at PSI Ring Cyclotron

**red: tracking simulation [OPAL]**
**black: measurement**

**dynamic range:**
factor 2.000 in particle density

**position of extraction septum**
\( d = 50\mu m \)

**turn numbers from simulation**

[Y.Bi et al]
RF acceleration

- acceleration is realized in the classical way using 2 or 4 “Dees”
- or by box resonators in separated sector cyclotrons
- frequencies typically around 50…100MHz, harmonic numbers \( h = 1...10 \)
- voltages 100kV…1MV per device

RF frequency can be a multiple of the cyclotron frequency:

\[
\omega_{RF} = h \cdot \omega_c
\]
RF and Flattop Resonator

for high intensities it is necessary to flatten the RF field over the bunch length
→ use 3rd harmonic cavity to generate a flat field (over time)

optimum condition: $U_{tot} = \cos \omega t - \frac{1}{9} \cos 3\omega t$

broader flat region for bunch
longitudinal space charge

sector model (W.Joho, 1981):
→ accumulated energy spread transforms into transverse tails
• consider rotating uniform sectors of charge (overlapping turns)
• test particle “sees” only fraction of sector due to shielding of vacuum chamber with gap height $2w$

two factors are proportional to the number of turns:
1) the charge density in the sector
2) the time span the force acts

\[
\Delta U_{sc} = \frac{8}{3} e I_p Z_0 \ln \left( 4 \frac{w}{a} \right) \cdot \frac{n^2_{\text{max}}}{\beta_{\text{max}}} \approx 2.800 \Omega \cdot e I_p \cdot \frac{n^2_{\text{max}}}{\beta_{\text{max}}}
\]

derivation see: High Intensity Aspects of Cyclotrons, ECPM-2012, PSI

in addition:
3) the inverse of turn separation at extraction:
\[
\frac{1}{\Delta R_{\text{extr}}} \propto n_{\text{max}}
\]

▶ thus the attainable current at constant losses scales as $n_{\text{max}}^{-3}$
longitudinal space charge; evidence for third power law

• at PSI the maximum attainable current indeed scales with the third power of the turn number
• maximum energy gain per turn is of utmost importance in this type of high intensity cyclotron

→ with constant losses at the extraction electrode the maximum attainable current indeed scales as:

$I_{\text{max}} \propto n_t^{-3}$
different regime for very short bunches: formation of circular bunch

**in theory**
strong space charge within a bending field leads to rapid cycloidal motion around bunch center
[Chasman & Baltz (1984)]
→ bound motion; circular equilibrium beam distribution
→ see Ch.Baumgarten, ECPM 2012, PSI

**in practice**
time structure measurement in injector II cyclotron → circular bunch shape observed

in simplified model: test charge in bunch field with vertically oriented bending field

blowup in ~20m drift
vertical force from space charge: \[ F_y = \frac{n_v e^2}{\epsilon_0 \gamma^2} \cdot y, \quad n_v = \frac{N}{(2\pi)^{\frac{3}{2}} \sigma_y D_f R \Delta R} \]

focusing force:

\[ F_y = -\gamma m_0 \omega_c^2 \nu_y^2 \cdot y \]

thus, eqn. of motion:

\[ \ddot{y} + \left( \omega_c^2 \nu_y^2 - \frac{n_v e^2}{\epsilon_0 m_0 \gamma^3} \right) y = 0 \]

→ equating space charge and focusing force delivers an **intensity limit for loss of focusing**!

tune shift from forces:

\[ \Delta \nu_y \approx -n_v \frac{2\pi r_p R^2}{\beta^2 \gamma^3 \nu_y^2} \]

\[ \approx -\sqrt{2\pi} \frac{r_p R}{\epsilon \beta c \nu y_0 \sigma_z} \frac{m_0 c^2}{U_t} I_{avg} \]
Beam dynamics modeling for high intensity beams in cyclotrons – general comments

**Multiscale / Multiresolution**
- Maxwell's equations in 3D or reduced set combined with particles; large and complex structures (field computations)
- many particles problem, $n \sim 10^9$ per bunch in case of PSI
- Spatial scales: $10^{-4} \ldots 10^4 m \rightarrow O(1E5)$ integration steps; advanced numerical methods; parallel computing
- neighboring bunches (Cyclotrons & FFAG)

**Multiphysics**
- particle matter interaction, simulation of scattering
- field emission in resonators
- secondary particles

at PSI development of **OPAL** code with many extensions in recent years
see: amas.web.psi.ch

[A.Adelmann]
parallel computing - scalability

scaling test:
- grid 1024x1024x1024
- particles 1E9

- full 3D tracking run
- no parallel I/O considered in timing
- smallest number of cores = 1024 to run this problem

real case:
- 5E6 32 cores, 108 turns 64x128x64 + I/O \rightarrow 19 hours (modern cluster)
examples of OPAL simulations in PSI Ring

distribution with **varying initial length** after 100 turns → short bunch stays compact, no tails!

tracking with 0, 6, 8 neighboring bunches;
considered bunch shows slight compression when taking neighbours into account [J.Yang, A.Adelmann]
Outlook: Cyclotrons II

• cyclotron subsystems
  extraction schemes, RF systems/resonators, magnets, vacuum issues, instrumentation

• applications and examples of existing cyclotrons
  TRIUMF, RIKEN SRC, PSI Ring, PSI medical cyclotron

• discussion
  classification of circular accelerators, cyclotron vs. FFAG, Pro’s and Con’s of cyclotrons for different applications